APPLICATION OF THE BINOMIAL PROBABILITY DISTRIBUTION AND DISCRIMINATION NET THEORY IN MODELING SHORT-TERM HUMAN MEMORY IN A CONSOLE OPERATOR SITUATION

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THESIS

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
June 1973

ABSTRACT

Recall of paired-associate, binary-coded, three-digit numbers was tested in an effort to determine the applicability of the binomial probability distribution in modeling short-term, human memory. Further, the use of discrimination net theory principles in tests of recall was examined. Although the analysis showed the binomial probability distribution to be applicable in modeling memory to the 0.95 level of significance, by no means is the model capable of universal application; in fact, its application is probably as localized as the assumptions made about it and its functional frame of reference - that of a console operator.

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I. BACKGROUND

The history of the formal study of human memory is rich and varied. It is possible to trace a written record of speculation about this aspect of the human thought process to the ancient Greek philosophers, and the dedicated scholar could, no doubt, discover even earlier records. As early as 86 - 82 B.C., the Romans had a text of techniques for improving an individual's memory entitled, Rhetorica ad Herennium (Yates, 1966). In fact, the principles given in Rhetorica ad Herennium for establishing an "artificial" memory are still used by modern practitioners in the field of memory.

To the Greeks and Romans, rhetoric was an important part of life. Today, with the convenient access to libraries, there is little need to memorize books, poems, or other literature. Hence, the art of memory has fallen into disuse.

Although modern science has added little to man's knowledge of the practical art of remembering, modern man's curiosity of the basis of memory has greatly increased. Current theories in the field of study known as verbal learning and memory utilize a synthesis of two major fields of science, physiology and psychology. It is this blend that has enabled the experimental psychologists to accomplish pioneering work in an effort to understand the innermost arcana of the human mind.

A. PHYSIOLOGICAL

Guyton (1971), in his Textbook of Medical Physiology, has formulated a definition of a thought in terms of neural activity as follows:

> A thought probably results from the momentary "patterns" of stimulation of many different parts of the nervous system

> at the same time, probably involving most importantly the cerebral cortex, thalamus, rhinencephalon, and the upper reticular formation of the brain stem. The stimulated areas of the rhinencephalon, thalamus, and the reticular formation probably determine the crude nature of the thought, giving it such qualities as pleasure, displeasure, pain, comfort, crude modalities of sensation, localization to general parts of the body, and other gross characteristics. On the other hand, the stimulated area of the cerebral cortex probably determines the discrete characteristics of the thought (such as specific localization of sensation in the body and of objects in the fields of vision), discrete patterns of sensation (such as the rectangular pattern of a concrete block wall or the texture of a rug), and other individual characteristics that enter into the overall awareness of a particular instant.

Guyton further defines memory as "the capability of recalling a thought at least once and usually again and again."

It is obvious from the above definitions, that the mechanism of memory is equally as complex as the mechanism of a single thought, for to provide memory, the nervous system must create the identical spatial and temporal pattern of stimulation of a single thought in the central nervous system at a future date.

Although memory cannot be explained and substantiated in any great detail, physiologists do have an understanding of the basic neuronal processes that lead to memory. Most physiologists distinguish three types of memory, based on length of retention. For the purposes of this paper, memory will be divided into (1) instantaneous memory, one's ability to recall information from second to second, and (2) fixed memory, which includes two major sub-divisions: (a) short-term memory that lasts up to a "few days," and (b) long-term memory that can last up to a lifetime.

At this point, it is noteworthy to confer justification upon the separation of fixed memory into two divisions, short-term memory and long-term memory. Researchers holding a traditional inclination have

argued against the dichotomization of memory (i.e., Melton, 1963; Postman, 1964). Their argument is that one memory is simpler than two; but, the opposing camp contends that the inverse is usually the case. Often cited is a comparison of the model for free recall proposed by Atkinson and Shiffrin (1968) and the model proposed by Postman and Phillips (1965). Here the contention is made that, "Any single process system making a fair attempt to explain the mass of data currently available must, of necessity, be sufficiently complex that a single process becomes a misnomer." (Atkinson and Shiffrin, 1968).

Perhaps the single most convincing demonstration of the requisite dichotomy of memory is Milner's report (1966, Abstract) on the effects of hippocampal lesions. In part, it states,

> Bilateral surgical lesions in the hippocampal region, on the mesial aspect of the temporal lobe, produce a remarkably severe and persistent memory disorder in human patients, the pattern of breakdown providing valuable clues to the cerebral organization of memory. Patients with these lesions show no loss of pre-operatively acquired skills, and intelligence as measured by formal tests is unimpaired, but, with the possible exception of acquiring motor skill, they seem largely incapable of adding new information to the long-term store. This is true whether acquisition is measured by free recall, recognition, or learning with savings. Nevertheless, the immediate registration of new input (as measured, for example, by digit span and dichotic listening tests) appears to take place normally and material which can be encompassed by verbal rehearsal is held for many minutes without further loss than that entailed in the initial verbalization. Interruption of rehearsal, regardless of the nature of the distracting task, produces immediate forgetting of what went before, and some quite simple material which cannot be categorized in verbal terms decays in 30 seconds or so, even without an interpolated distraction. Material already in long-term store is unaffected by the lesion, except for a certain amount of retrograde amnesia for preoperative events.

Apparently, a short-term store remains in the patients, but the aforementioned lesions have produced a breakdown either in the ability to store new information in long-term store or to retrieve new

information from it. These patients appear to be incapable of retaining new information on a long-term basis.

A related defect, known as Korsakoff's syndrome, also lends credence to the concept of memory dichotomy. Patients with this disorder are unable to retain new events for longer than a few seconds to minutes, but their memory of events and people prior to their illness remains intact.

As with most clinical research, there are several points to be considered before drawing a conclusion. First, the patients observed were in a general sense, abnormal to begin with; second, once the memory defects were discerned, the operations were discontinued, thus leaving only a small number of patients to observe; and third, the results of the lesions were variable, seemingly dependent upon the size of the lesion, larger lesions giving rise to the full syndrome.

As compelling to the argument of the dichotomy of memory as the patients were, the above criticism may lead to discounting them as anomalies. Thus, further experiments were conducted which added to the acceptance of Milner's observations. Patients who had known damage to the hippocampal area in one hemisphere of the cerebrum were tested for memory deficit after an injection of sodium amytal which inactivated the normal hemisphere. Control groups consisted of patients with known lesions, and patients who received injections inactivating the damaged hemisphere. A number of memory tests were administered as a criterion to judge memory deficit. One such test consisted of the presentation of four pictures, followed by a distraction, and a re-presentation of the four pictures but this time in a set of nine. If the patient was unable to identify the four critical pictures, then memory deficit was assumed.

The results showed that in all cases, memory deficit occurred only after bilateral damage; that is, if side A is damaged and side B is inactivated then memory deficit appears, but if the inactivated side is the damaged side also, then no deficit occurs. These additional tests tend to indicate that the cases Milner described were not anomalies and therefore tend to lend strong support to the contention that there exists a distinction between short-term and long-term memory.

If a memory is to last in the brain, it must become "fixed" in the neuronal circuits. This process requires 15 to 20 minutes for minimal fixation and at least an hour for maximal fixation. As an example, if a sensory impression is made on the brain but is followed by strong electrical stimulation or electroconvolusive shock of the brain, no sensory experience will be recalled. If however, the same sensory impression is permitted to remain for an hour, the memory engram will be established and the sensory impression will be remembered.

In an effort to explain this phenomenon, the neuropsychologist, D. O. Hebb, has proposed the reverberating circuit theory of memory (Hebb, 1949). Briefly, what this theory advocates is the adoption of the concept of reverberating circuits within the cerebral cortex and thalamus portions of the cerebrum. For example, when a tetanizing electrical stimulus is directly applied to the surface of the cerebral cortex and then removed after a second or so, the local area excited by this stimulus continues to emit rhythmic action potentials for minutes, or under favorable conditions, for as long as one hour. This result, it is contended, results from local reverberating circuits, the impulses passing through a multiconnected circuit of neurons in the local area of the cortex itself or back and forth between the cortex and the thalamus.

It is presumed that sensory signals reaching the cerebral cortex can establish similar reverberating oscillations that continue for a matter of minutes. Then, as the reverberating circuits fade and the oscillations cease, the temporary memory fades.

One of the principle observations in support of this theory of temporary memory is that any factor that causes a general disturbance of brain function immediately erases all temporary memory, and the memory cannot be recalled following the disturbance.

Two other theories are closely allied in that they both deal with the potentiation aspect of neuronal processes. First, consider the post-tetanic potentiation theory. In most areas of the nervous system, including the anterior motoneurons of the spinal cord, tetanic stimulation of a single neuron for a few seconds causes increased excitability of the neuron for periods of up to a few hours. If during this period, the neuron is re-stimulated, it responds much more vigorously than in the normal state, a phenomenon referred to as post-tetanic potentiation. The associated theory asserts that memory depends on the excitability of the neurons involved.

Another change that occurs in neurons following a period of excitation is a change in the membrane potential of the neuron, lasting from a matter of seconds to hours. Because this change alters the excitability of the neuron, it is contended to be the basis for memory. These changes in membrane potential are known as D.C. potentials. Measurement of such potentials in the cerebral cortex, especially in the dendritic layers, is confirmed.

Further discussion of memory, in physiological terms, will be truncated at this point because of the necessity to be converse with

the more specialized aspects of neuronal physiology. For the record, a reference is made to the following theories (Guyton, 1966):

1. The Theory of Alteration of Transmission Facility at the Synapse
2. The Glial Theory and Other Extraneuronal Theories
3. The Molecular Memory Theory
4. The Biopolyelectrolytic Model of Memory (Katchalsky and Neumann, 1972)

B. PSYCHOLOGICAL

An overview of recent trends in the field of study concerning human verbal learning and memory indicates that a basic change has occurred over the last decade in that there has been a transposition of the unit of analysis. The current unit of analysis is the single test item, rather than the entire test list that had been used in the past. Ten years ago, memory studies went under the title of "verbal learning," and the routine measure of performance was the number of trials to criterion value. Retention measures were expressed in terms of the total number of correct responses or the total number of errors after a specified retention interval.

Currently, the basic datum in memory studies is the probability of correct recall or retention. Analysis focuses on the individual item, not the entire test list. This adoption of single items as the unit of analysis has re-oriented the field of memory study.

As a result of the shift in the unit of analysis, new concepts, assumptions, models, and experimental techniques have evolved. Twenty years is a comparatively short time span in light of the fact that the first major work in the field of memory was the contribution of

H. Ebbinghaus, Memory: A Contribution to Experimental Psychology, published in 1885. But, the changes have been considerable during the past two decades.

Ten years ago, it was asserted that, "Interference theory occupies the unchallenged position as the only major significant analysis of the process of forgetting." (Postman, 1961). Today, the emphasis is more on the encoding, storage, and retrieval processes of memory - and because of the plethora of existing models, there is no one unchallenged theory.

At this point, several major contributions which brought about the aforementioned change in the study of human memory may be noted. The first is the filter theory of D. E. Broadbent, formulated in 1958. Actually, Broadbent's theory was based on the thesis work of the English psychologist, John Brown.

In 1955, Brown suggested that memory, for periods of greater than a few seconds of duration, could be handled by a combination of a limited capacity memory system and an antecedent buffer storage system. If a series of items were input from the buffer to the limited capacity system (short-term memory), and if the latter then delivered the information back to the buffer storage, a limited number of items could be kept in circulation indefinitely. Each item must remain in the buffer storage long enough for all the other items to be passed through the limited capacity system that is utilized in perception. Thus, if any new action is required, the recirculation, commonly referred to as rehearsal, of the items in an individual's memory must be interrupted to deal with the newly presented incoming signals. Consequently, if memory of a particular item is to be preserved, the external signal must be ignored. Thus, memory working under this assumption, would be extremely

vulnerable to any intruding or external activity carried out during the retention/rehearsal period. Gradual transfer of the information from this recirculatory type of storage to a more permanent long-term store would then occur. The preceding argument concerning long-term store was not stressed by Brown, but it would appear to be a logical extension of his theory.

Thus, working under Brown's theory, Broadbent formulated the filter theory of memory. Broadbent's model adopted Brown's framework, but the impact of his work was in his argument that the human organism should be viewed as an information processing system - an argument so compelling that it is presently one of the most popular points of view held by psychologists dealing with human memory. An outline of the information processing system of human memory is presented in Figure 1.

The aforementioned system is represented in the form of an information flow diagram. When a stimulus is input to the system, it goes into a brief sensory buffer store, where rapid decay occurs unless the stimulus information is attended to (Brown, 1955). After the individual attends to the stimulus, it progresses to what is known as the short-term store. Material in the short-term store is then rehearsed or recirculated whereupon it enters the long-term store. Note that the information may be forgotten or lost from only the sensory buffer store or the short-term store.

From the diagram it is obvious that the short-term store corresponds to short-term memory and the long-term store is analagous to long-term memory, but what of the sensory buffer store? Instantaneous memory is conceived of as a buffer storage mechanism, enabling decoupling, for a minute amount of time, of the sensory processes from

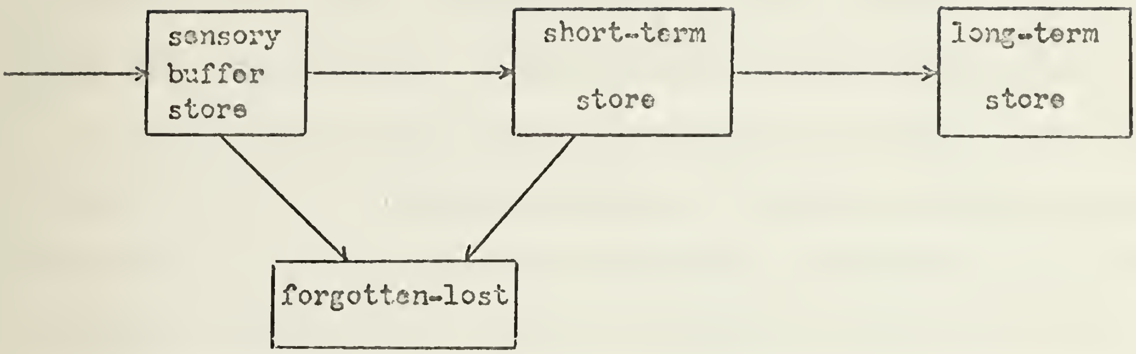

Figure 1. AN INFORMATION PROCESSING MODEL OF MEMORY

the central memory processes. The necessity of a sensory buffer store is seen in that the central memory processes are proceeding at their own rate and are not interruptable on an instant-to-instant basis to allow the memory process to pay attention to every millisecond's worth of sensory input. Therefore, a buffer is needed to explain how the central memory processes are able to complete a phase of their activity before having to go back and attending to the information in the buffer store. Thus, the sensory buffer store is seen as a decoupling and storage process between the input sensory processes and the central memory processes. Note that this is merely a refined version of Brown's contention.

A second main factor, this one experimental in nature rather than theoretical, was the detractor technique of testing short-term memory. This method of experimentation was developed concurrently by John Brown in England (1958), and M.J. and L.R. Peterson in the United States (1959). This technique brought short-term memory under experimental control in the laboratory and was able to reproduce, in a matter of seconds, the forgetting that had previously taken months to observe. Basically, this technique involves presentation of a stimulus, distraction of some sort, and subsequent testing of recall. The technique is described in "Short-term retention of individual verbal items," (Peterson, 1959).

Thirdly, A.W. Melton's (1963) "Implications of short-term memory for a general theory of memory," placed the new work being done in short-term memory in the limelight. This influential paper interrelated the traditional field of verbal learning to the upcoming theories of short-term memory, and thus heightened the interest in memory studies.

While these are the major contributions that effected the change in experimental psychology's field of memory study, they are by no means

the only works of influence. Other main contributors include: Sperling's work on iconic memory; Waugh and Norman's distinction between primary and secondary memory; Bower's multicomponent model of memory; Conrad's work on acoustic confusions; Mandler and Tulving's work on organizational and retrieval processes; and Murdock, Norman, and Wickelgren's application of signal theory to memory (Murdock, 1972).

II. THE EXPERIMENT

With the preceding background and the adoption of an information processing point of view of the human memory, an experiment was devised in an attempt to develop a human memory modeling technique. The basic components of the experiment were drawn from information theory, interference theories of memory, the binomial probability distribution, and the discrimination net theory of paired-associate learning.

A. BACKGROUND

1. Information Theory

The impact of information theory in the field of human memory was described in the previous section, and its impact in this experiment will be seen in the equations in the section delineating the theories involved in the experiment.

2. Interference Theory

During the 1950's, the understanding of forgetting in long-term memory was considerably advanced. In particular, the analysis of the mechanism by which one memory could suffer interference from other memories was concentrated upon. This mechanism had been shown to account for a majority of forgetting, and it was questionable whether or not long-term memory any longer required the idea that some engram or trace of the original stimulus decayed gradually with time in the absence of interfering memories. For this reason, interference theorists tended to resist the idea of simple decay over time even in short-term memory, and began to argue that the same principles established in long-term memory were applicable to short-term memory as well.

A full analysis of forgetting through interference is much too complex to review completely. Thus, reference is made to reviews done by Postman, (1961).

It is, however, possible to present a brief, simplified account of the advances of the 1950's in the field of memory. According to interference theory, the learning of an association between a particular stimulus and response, S_1 and R_1, can be interfered with by a subsequent learning of some other stimulus-response pair, S_2 and R_2. The obvious mechanism of the interference is the similarity of stimuli, so that in fact a "common" stimulus, S, was being attached first to R_1, then to R_2. Suppose then that only one response can occur. There are two types of interference that can occur; first, performance of the first response, R_1, would be poor after the learning of the second response, R_2. This type of interference is known as retroactive inhibition. Secondly, the learning of R_2 might be worse than it would have been without learning R_1. This concept is known as proactive inhibition.

Experimentally, both retroactive and proactive inhibition can be demonstrated and it is through this type of experimentation that Melton made a fundamental discovery in interference theory. In his classic work of 1940 with Irwin and 1941 with Lackum, Melton found that retroactive inhibition is not a simple transfer of a new response to supersede the original one. If the degree of practice given to R_2 is varied, the amount of interference shown by testing R_1 is also varied. The more learning of R_2 that is undertaken, the worse the performance on R_1. This seems to be a logical finding and to be expected. However, this retroactive inhibition occurs not only because the subject demonstrates R_2 when he is tested for R_1, but also because the number of intrusion errors rises as practice on R_2 increases to a moderate level,

then falls off as practice on R_2 continues. Consequently, there is part of the forgetting of R_1 that can be ascribed to the intrusion errors of R_2, and a further part which continues to appear even when R_2 has been practiced to the point wherein intrusions become rare. Thus, if a subject is taught R_1, then given a great deal of practice on R_2, and subsequently tasked with the relearning of R_1, he will do very poorly at first then improve to a high level of performance.

With this in mind, if one focuses on the studies of short-term memory in the 1960's, it is obvious that almost all research involved interference theory. Rapid forgetting, which is evidence of a fading trace, can only be shown if some interfering activity is inserted following the presentation of material to be remembered. Consonant with Broadbent's views, this latter activity occupies the limited capacity system and halts rehearsal; but, equally, on the interference theory level, it could be regarded as retroactive inhibition.

Retroactive inhibition was emphasized in this section because it is the concept that will be utilized in the experiment.

3. The Binomial Probability Distribution

A fundamental class of problems involve random processes for which there are only two possible outcomes. The outcomes can occur without any fixed pattern, and the probability of either outcome remains unchanged for each trial. Processes with these characteristics are known as Bernoulli processes.

In order to simplify the expression of the characteristics of Bernoulli processes, a specified or desired outcome of a trial is termed a success. The number of ways in which r successes can be obtained in n trials is given by $\binom{n}{r}$. The generalized expression of the function

describing the probability of obtaining exactly r successes in n trials of an experiment is,

$$P(r;n,p) = \binom{n}{r} p^r q^{n-r}$$

and is known as the binomial probability distribution.

The rationale behind selection of the binomial probability distribution as the underlying distribution for the experiment is one of simplification. The discrete parameter binomial probability distribution was chosen over its continuous probability distribution counterparts because of its simplicity, ease of computation of data, and most importantly, because of its ease of application to interference theory memory experiments.

4. Discrimination Net Theory

In light of the psychologist's penchant to view memory as an information processing system, an interesting hypothesis was formulated by H.A. Simon and E.A. Feigenbaum in their work with the EPAM model, an information processing model (Simon and Feigenbaum, 1964). The idea is similar in concept to the memory systems of computers, which have a particular format of storage. The basic idea, as applied to stimulus-response or paired-associate studies, is that what gets stored with the stimulus is not a representation of the response itself but merely information about how to locate that response within a discrimination net existing in the memory stores. In other words, what is stored or associated with the stimulus is a program of instructions which will enable the retrieval of the correct response.

A discrimination net, sometimes called a sorting tree, is a notion which states that the classification of inputs is accomplished by making a series of decisions about the input. The net or tree

consists of a sequence of test nodes and branches; each node testing for a characteristic of the input, each branch leading to another test node. For example, consider a binary discrimination net as in Figure 2. The test nodes ask whether or not there is a 1 in each of the three positions of the three digit number (input). Depending upon whether the answer is yes (+) or no (-), the retrieval is deflected to the right or left. Paired-associate learning refers to a stimulus and its associated program of retrieval of response. By following the retrieval program associated with a stimulus, the correct response may be located. Consider the number 101, enter the discrimination net, answer the test node questions, and indeed the number 101 will be located.

B. MODEL

The theory to be applied is as follows:

The response contains N = (1 or 3) bits of information according to the information theory formula,

$$N = \log_2 L$$

where,

N = the amount of information the subject must process, in bits

L = the number of alternatives the subject has

For a given value of t interpolations, let $r(t)$ be the probability that a single bit of information is retained. Note that an interpolation is defined to be an intervening stimulus-response pair or paired-associate item between the initial presentation of a paired-associate item and its test for recall. Let $R_N(t)$ be the number of retained bits of information after t interpolations, given that N bits were originally input. $R_N(t)$ has a binomial probability distribution given by,

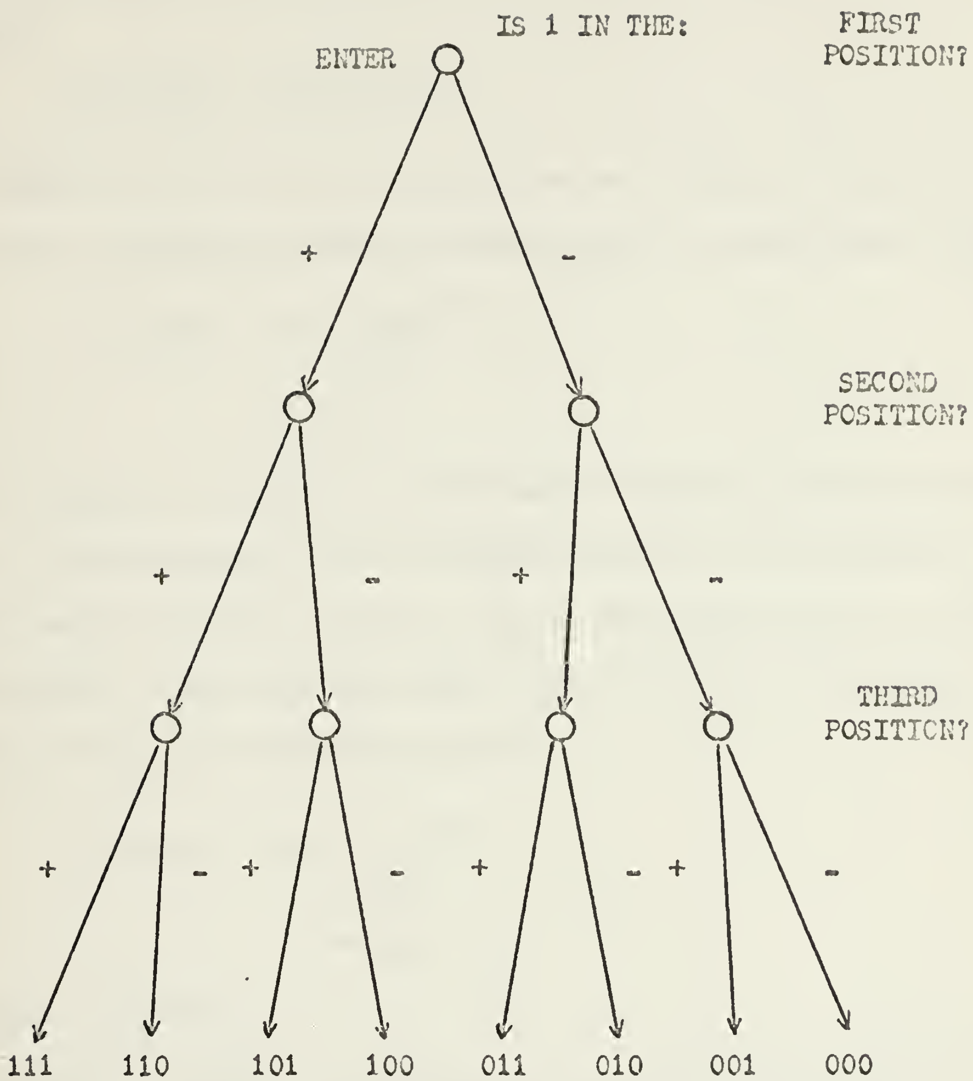

Figure 2. BINARY DISCRIMINATION NET. Each node requires a binary choice that, in three stages, leads to a required response.

$$P(R_N(t) = x) = \binom{N}{x} r(t)^x \, (1-r(t))^{N-x}$$

For each bit of information lost (not retained), the probability of a correct response is reduced by 0.5. Letting C be the event of a correct response,

$$P(C : R_N(t) = x) = 0.5^{N-x}$$

For example, if N = 3 bits of information were originally input, and x = 3 bits of information were retained after t interpolations, then,

$$\begin{aligned} P(C : R_N(t) = 3) &= 0.5^{3-3} \\ &= 0.5^0 \\ &= 1 \end{aligned}$$

which is logical for if all the information originally input is retained after t interpolations, then the subject should be 100% certain of the correct response (target number). If, on the other hand, N = 3 bits of information were originally input, but x = 0 bits of information were retained after t interpolations, then

$$\begin{aligned} P(C : R_N(t) = 0) &= 0.5^{3-0} \\ &= 0.5^3 \\ &= 0.125 \end{aligned}$$

as would be expected.

Now, from the above two equations, the unconditional probability of a correct recall may be derived as,

$$\begin{aligned} P(C_N(t)) &= \sum_{x=0}^{N} P(C : R_N(t) = x) \cdot P(R_N(t) = x) \\ &= \sum_{x=0}^{N} 0.5^{N-x} \binom{N}{x} r(t)^x \, (1-r(t))^{N-x} \end{aligned}$$

where,

$P(C_N(t))$ = the probability of a correct response after t interpolations given that N bits were originally input

The parameters of this equation are $r(t)$ for $t = 1, 2, 3, 4, 5, 6, 7, 8$. The problem arises in determining what values to use for $r(t)$. This problem was reconciled by reviewing the data for the situation in which there were only two alternatives, $N = 1$ bit. Since $r(t)$ was defined as the probability that a single bit of information was retained after t interpolations, the data for $N = 1$ bit was utilized. For $N = 1$ bit,

$$
\begin{aligned}
P(C_N(t) &= P(C_1(t)) \\
&= \sum_{x=0}^{N} 0.5^{N-x} \left(\binom{N}{x} r(t)^x (1-r(t))^{N-x}\right. \\
&= 0.5^{1-0} \binom{1}{0} r(t)^0 (1-r(t))^{1-0} + \\
&\quad 0.5^{1-1} \binom{1}{1} r(t)^1 (1-r(t))^{1-1} \\
&= 0.5^1 \binom{1}{0} r(t)^0 (1-r(t))^1 + \\
&\quad 0.5^0 \binom{1}{1} r(t)^1 (1-r(t))^0 \\
&= 0.5\,(1-r(t)) + r(t) \\
&= 0.5 - 0.5\,r(t) + r(t) \\
P(C_1(t)) &= 0.5 + 0.5\,r(t)
\end{aligned}
$$

Now, from the observed values of Percent Correct Responses versus t for $N = 1$ bit (2 alternatives), let $P(C_N(t))$, the unconditional probability of a correct response, equal the values obtained as percent correct response. Note that this can only be done for the situation wherein $N = 1$ bit. since the problem is to determine $r(t)$, the probability that a single bit of information is retained after t interpolations.

If the above assumption is made,

$$P(C_1(t)) = 0.5 + 0.5\,r(t) \Rightarrow r(t) = 2\,P(C_1(t)) - 1$$

Thus, by the above argument, the values of $r(t)$ may be determined.

Now, since the values of r(t) may be solved for, it is a simple matter to predict the values for $P(C_3(t))$, using the formula,

$$P(C_3(t)) = \sum_{x=0}^{3} 0.5^{N-x} \binom{N}{x} r(t)^x (1-r(t))^{N-x}$$

C. METHODS

1. Equipment

Two panels, each with eight miniature lamps and eight double throw toggle switches, were constructed and mounted on a base facing each other. One panel was used by the test administrator to present the stimuli of associated light-switch pairs corresponding to the recorded stimuli. The other panel was the subjects' panel for use in responding to the test administrator's stimuli. The panels were 30 inches long and 5 inches wide (Figure 3). The associated light-switch units on each panel were configured as shown in Figure 4 to emphasize the discrimination net possibilities.

A Sony cassette tape recorder was utilized in both recording and presenting the auditory stimuli (Figure 3).

A Lafayette 5040A timer was employed in the recording of the stimuli, in order to enable a uniform time interval between the 3-digit numbers used as stimuli.

2. Subjects

Six subjects were used as a sample population in the experiment; three males and three females. The age of the subjects ranged from 21 years to 23 years. The level of education of the subjects ranged from 2 years of undergraduate work for one female to graduate work for all of the males (U.S. Naval Postgraduate School). The other two females had

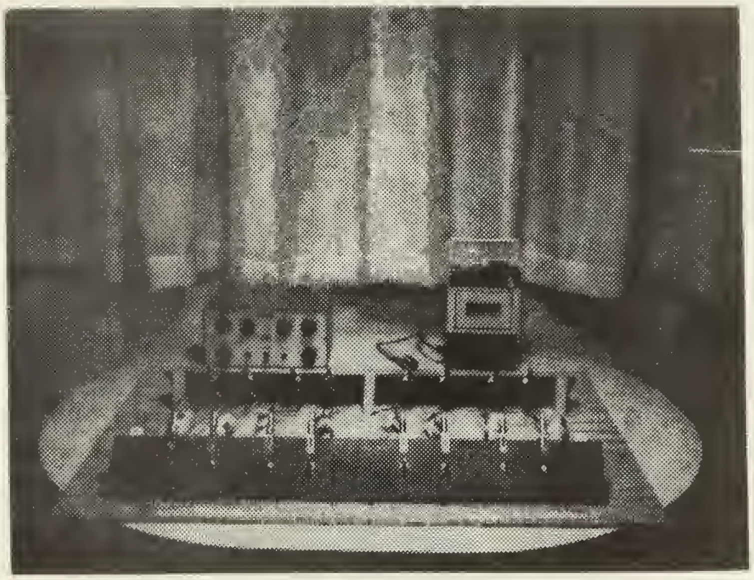

Figure 3. PHYSICAL EQUIPMENT OF THE EXPERIMENT. Administrator and subject panels, cassette tape recorder, and Lafayette 5040A timer.

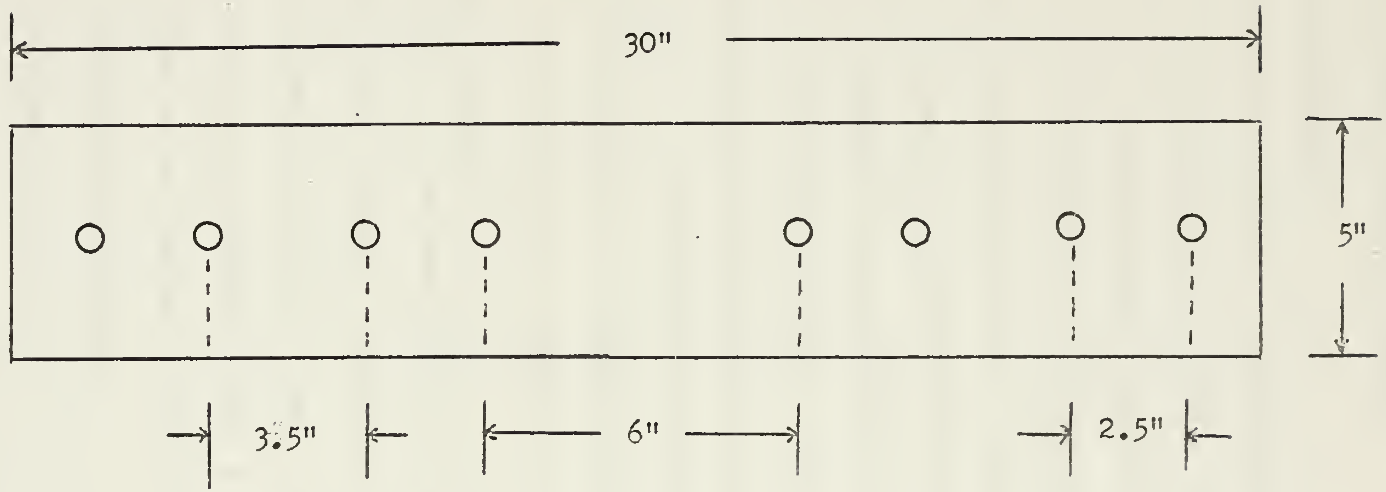

Figure 4. CONFIGURATION OF THE PANEL. Both administrator and subject have identical panels emphasizing discrimination net possibilities.

both completed undergraduate work. There was no criterion measure utilized in the selection of the subjects other than no history of participation in any memory building courses.

3. Experimental Design

The experiment involved a continuous stream of stimuli and tests of recall on single paired-associate items. There were three independent variables involved in the experiment, the first being the utilization or non-utilization of discrimination net principles in the arrangement of light-switch pairs on the test panels. Secondly, the number of interpolations or intervening paired-associate items presented between initial presentation and subsequent re-presentation of the target number being tested for was an independent variable. Third, the effect of the number of trials at each level of interpolation was an independent variable.

Thus, Phase 1 of the experiment utilized discrimination net principles in the arrangement of light-switch pairs on the test panels whereas Phase 2 disregarded such principles. Further, in both phases, the subjects were presented with from 1 to 8 interpolations between initial presentation and re-presentation of the target number. Finally, the subjects underwent 10 trials at each level of interpolation. Note that although the number of trials at each level of interpolation was fixed at 10, the data was recorded in such a way as to enable determination of the effect of any number of trials up to and including 10, upon the subjects' performance (Appendix E).

The six subjects underwent 10 trials at each of the 8 levels of interpolations in both phases under two different sets of alternatives. In one situation, the subject was told that the target number was associated with one of only two possible alternatives. It should be noted

here that the two possible alternatives differed from trial to trial and were presented immediately prior to testing. In the other situation, the subject was told that the target number was associated with any one of the 8 possible alternatives. The reasoning behind this dual alternatives situation was made evident in the MODEL section of the experiment; the $N = 1$ bit (2 alternatives) situation merely enabled computation of the theoretical values of $P(C_3(t))$. The entire experiment was conducted under the 2 alternatives situation then re-conducted under the 8 alternatives situation.

Thus, by utilizing 2 phases, each with 8 levels of interpolations, and 10 trials at each level of interpolation, the experiment was designed to be able to determine the effect of each independent variable upon the dependent variable, the subjects' performance.

4. Procedure

The experiment was conducted at the U.S. Naval Postgraduate School, Monterey, California. The six subjects chosen for the experiment were seated on the subjects' side of the apparatus, directly across from the test administrator. The subjects were told the nature of the experiment in that it was a test of memory. For purposes of familiarization with the equipment, several trials were conducted prior to the actual testing. The subjects were instructed to focus their attention on the panel before them and not be distracted by the hand movements of the test administrator. Note that during the actual test sessions, a screen was placed between the subjects' panel and the administrator's panel. Subjects were told that they were to be exposed to an auditory stimulus (three-digit, binary-coded number) and an associated light-switch pair to study for 3.5 seconds (arbitrarily chosen). Thus, when

an auditory number was presented, an associated light was illuminated on the subject's panel, and the subject was told to turn the light off using the switch directly below the light, and associate the light-switch unit with the number heard. After either 1, 2, 3, 4, 5, 6, 7, or 8 interpolations, the target number recurred alone (without a light), preceded by the word "test." The subjects were tasked with the problem of recalling that light-switch unit associated with the target number. The subject had 3 seconds in which to perform the task, being told to guess if necessary, before the next auditory stimulus was presented. Ten such stimulus-response pairs were presented at each level of interpolation and a record was kept of the subjects' successes on each trial (Appendix E).

The stimuli were three-digit, binary-coded numbers recorded every 3.5 seconds on a cassette tape recorder; i.e., 100, 101, 000, etc. The order and frequency of the numbers were determined through the use of a random number table.

The order of the experiment trials was as follows: testing under the $N = 1$ bit situation, phase 1; testing under the $N = 3$ bits situation, phase 1; testing under the $N = 1$ bit situation, phase 2; testing under the $N = 3$ bits situation, phase 2.

D. RESULTS

The data collected from the conduct of the experiment is as would be expected. The Percent Correct Responses for both phases shows a trend toward degeneration as a function of increased interpolation in each of the two different number of alternative situations, $N = 1$ bit and $N = 3$ bits (Table 1 and Figure 5).

Table 2 tabulates the values of $r(t)$ calculated from the equation,

$$P(C_1(t)) = 0.5 + 0.5\, r(t) \Longrightarrow r(t) = 2\, P(C_1(t)) - 1$$

The computation of values for $r(t)$ enabled the computation of the theoretical values for $P(C_3(t))$ (see Table 3 and Figure 6).

The theoretical values of $P(C_3(t))$ in Table 3 are compared against observed values of Percent Correct Responses in Table 1, in Figure 7 for phase 1 and Figure 8 for phase 2.

The effects of the number of interpolations upon the observed values of $P(C_3(t))$ in phase 1 and phase 2 are shown in Figure 9.

Review of the data for the Percent Correct Response for the $N = 3$ bits condition, the observed values of $P(C_3(t))$, and the computed values of the Percent Correct Responses for the $N = 3$ bits condition, the theoretical values of $P(C_3(t))$, (Table 3) for both phase 1 and phase 2 shows these values to be closely aligned (Figures 7 and 8). An observed sample distribution of any type may be compared to a theoretical distribution assumed to be the population distribution from which the sample is drawn. If there is a degree of conformity between the two distributions, any slight difference is attributable to sampling variation. If however, any large discrepancy exists between the two distributions, then it may be concluded that the sample was drawn from some distribution other than the theoretical distribution.

Under this premise, a chi-square goodness-of-fit test was performed upon the observed and theoretical values of $P(C_3(t))$ for phase 1 and phase 2. The obtained values of the chi-square in phase 1 was 5.913 and for phase 2, 36.553. The value of chi-square at the 0.05 level for a one-tailed test with eight degrees of freedom is 15.507. Consequently, H_0 was accepted in phase 1 and rejected in phase 2; only in phase 1 did the chi-square goodness-of-fit test demonstrate no evidence to discount the applicability of the binomial probability distribution as an underlying

Table 1

PERCENT CORRECT RESPONSES IN PHASE 1 AND PHASE 2
FOR THE N=1 BIT AND N=3 BITS CONDITIONS

Number of Interpolations	*Percent Correct Responses, Phase 1		*Percent Correct Responses, Phase 2	
	N=1 Bit	N=3 Bits	N=1 Bit	N=3 Bits
1	95.0	91.6	95.0	90.0
2	88.3	78.3	88.3	71.6
3	85.0	65.0	90.0	53.3
4	83.3	53.3	83.3	48.3
5	83.3	55.0	85.0	41.6
6	80.0	50.0	80.0	36.7
7	78.3	48.3	78.3	35.0
8	78.3	43.3	76.6	31.7

* Each percentage is based on 60 observations

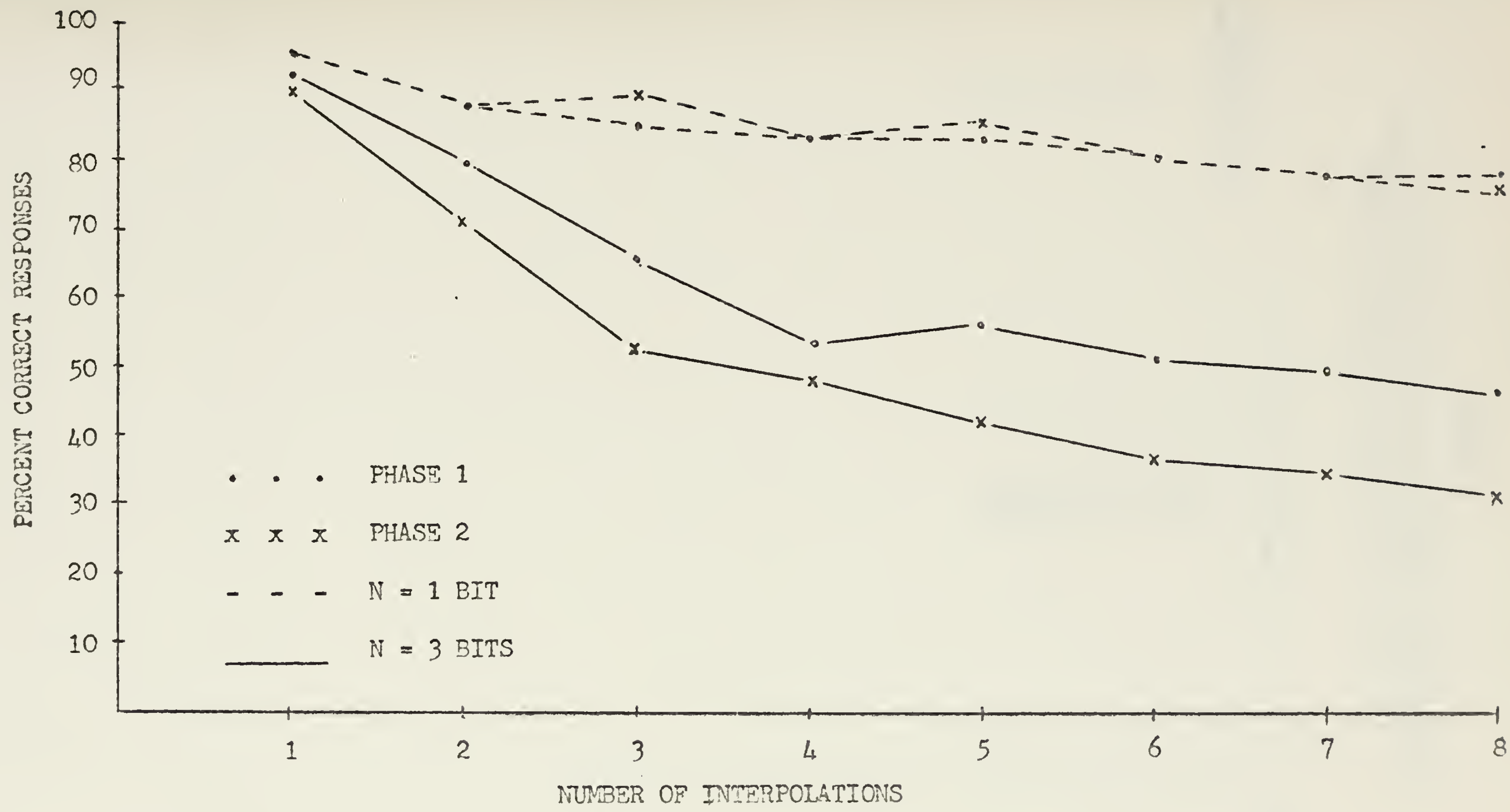

Figure 5. PERCENT CORRECT RESPONSES IN PHASE 1 AND PHASE 2. FOR THE N = 1 BIT AND N = 3 BITS CONDITION.

Table 2

TABULATED VALUES OF r(t) FOR PHASE 1 AND PHASE 2.
Derived from the equation for $P(C_1(t))$.

Number of Interpolations	r(t), Phase 1	r(t), Phase 2
1	.900	.900
2	.766	.766
3	.700	.800
4	.667	.667
5	.667	.700
6	.600	.600
7	.567	.567
8	.567	.532

Table 3

COMPUTED THEORETICAL VALUES OF $P(C_3(t))$
FOR PHASE 1 AND PHASE 2

Number of Interpolations	Theoretical $P(C_3(t))$ Phase 1	Theoretical $P(C_3(t))$ Phase 2
1	.857	.857
2	.689	.689
3	.614	.729
4	.579	.579
5	.579	.614
6	.512	.512
7	.481	.481
8	.481	.451

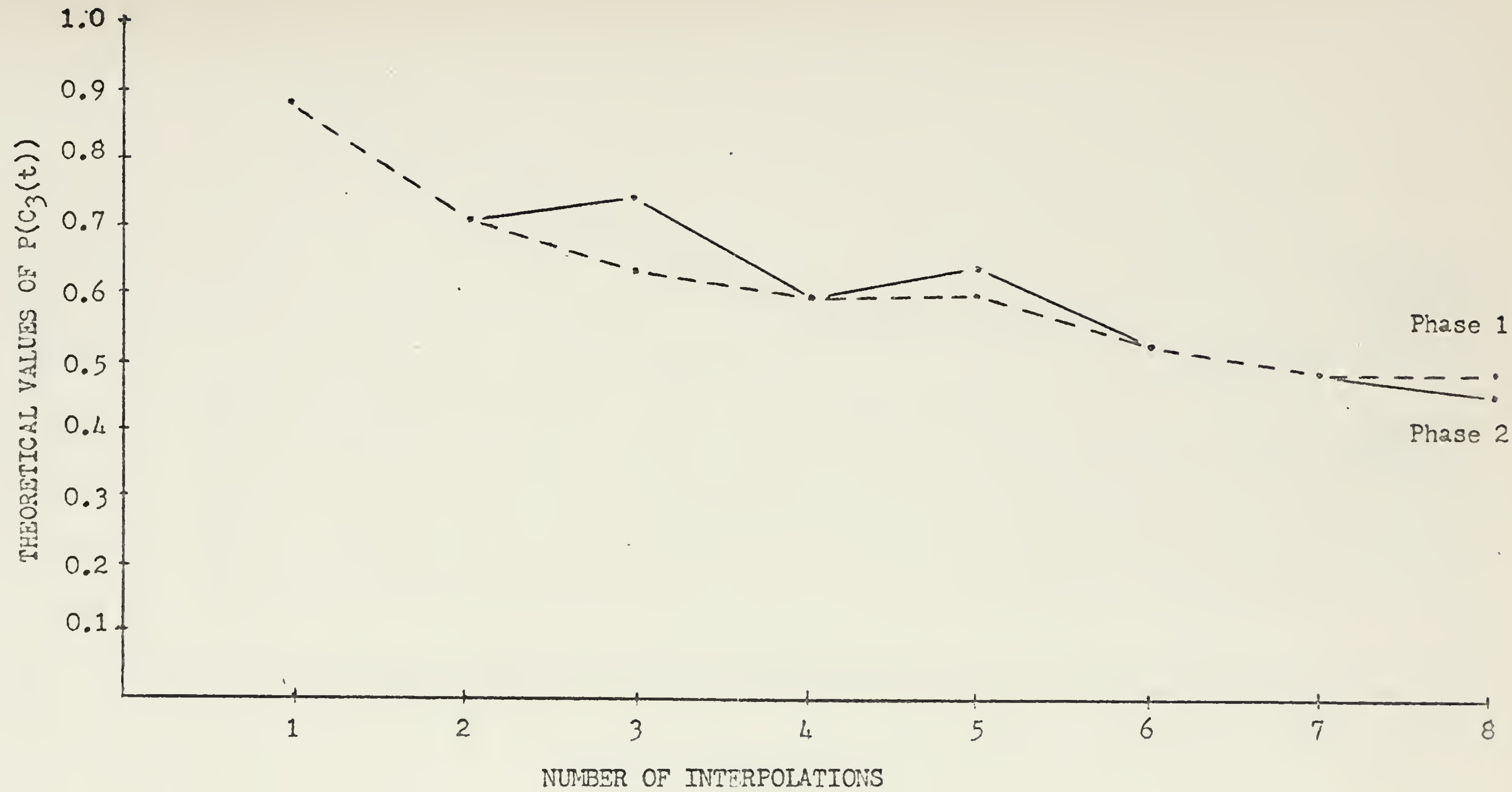

Figure 6. THEORETICAL VALUES OF $P(C_3(t))$.
Phase 1 versus Phase 2.

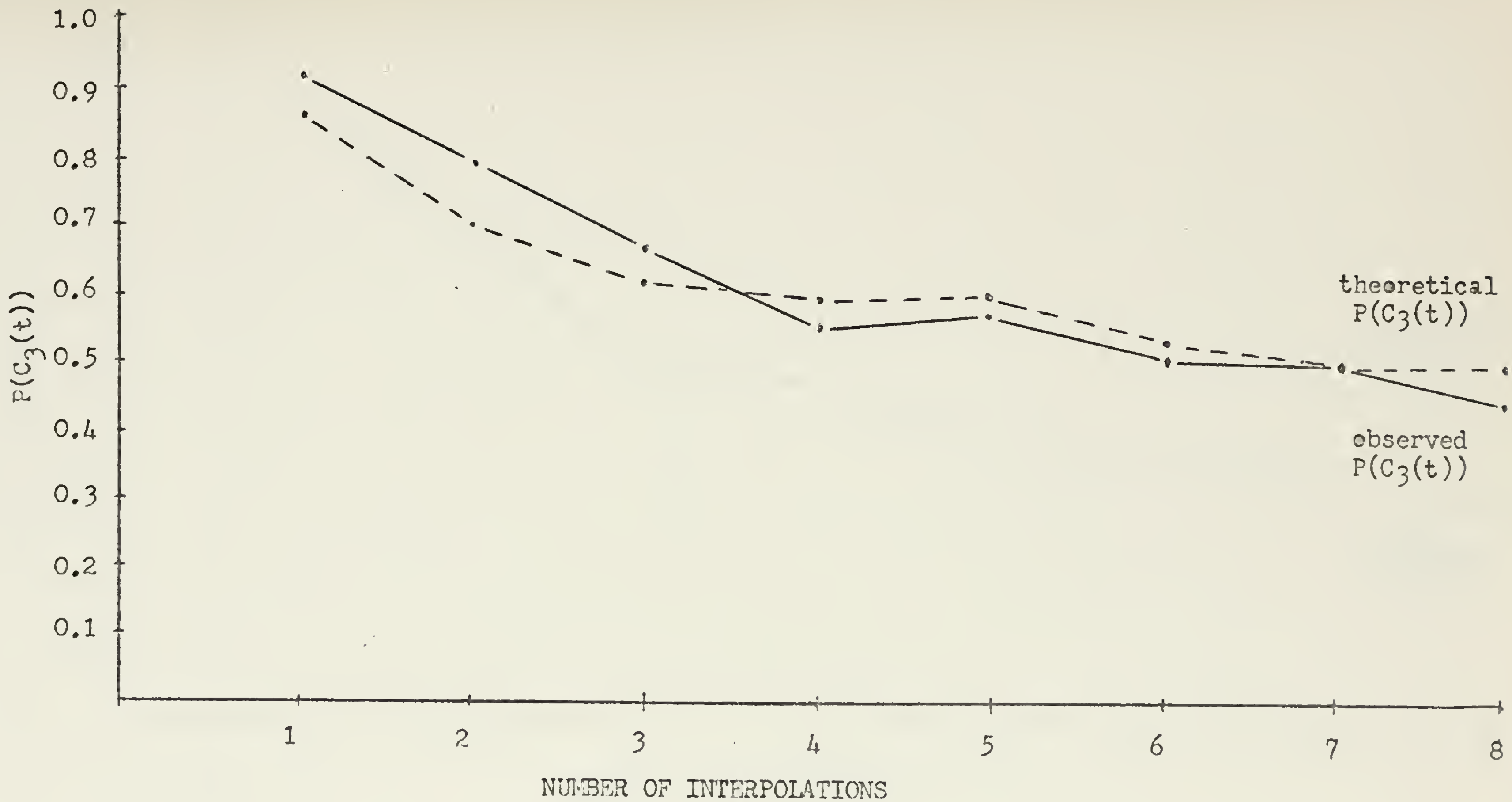

Figure 7. $P(C_3(t))$ VERSUS NUMBER OF INTERPOLATIONS FOR PHASE 1. Theoretical $P(C_3(t))$ versus Observed $P(C_3(t))$.

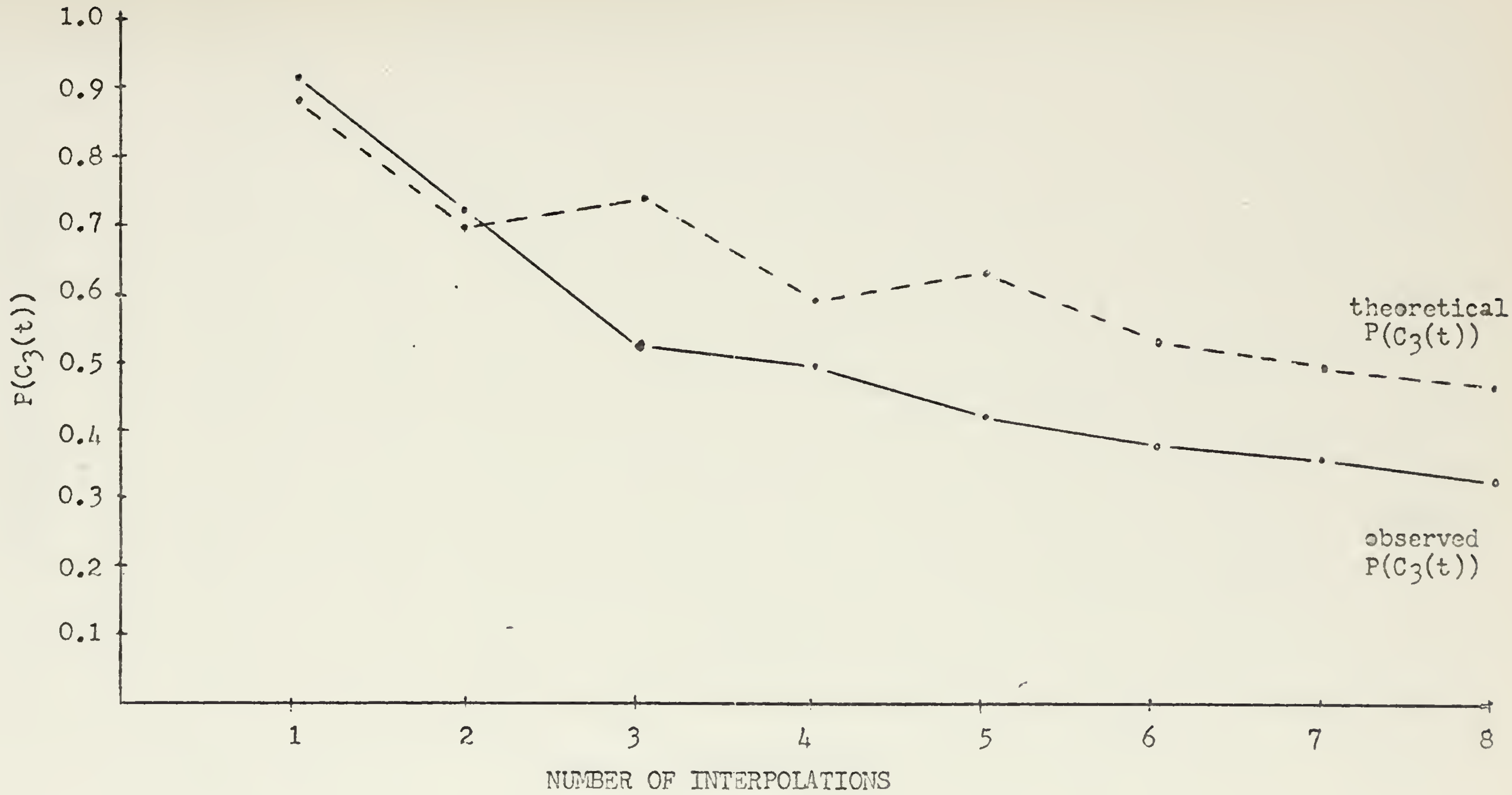

Figure 8. $P(C_3(t))$ VERSUS NUMBER OF INTERPOLATIONS FOR PHASE 2. Theoretical $P(C_3(t))$ versus Observed $P(C_3(t))$.

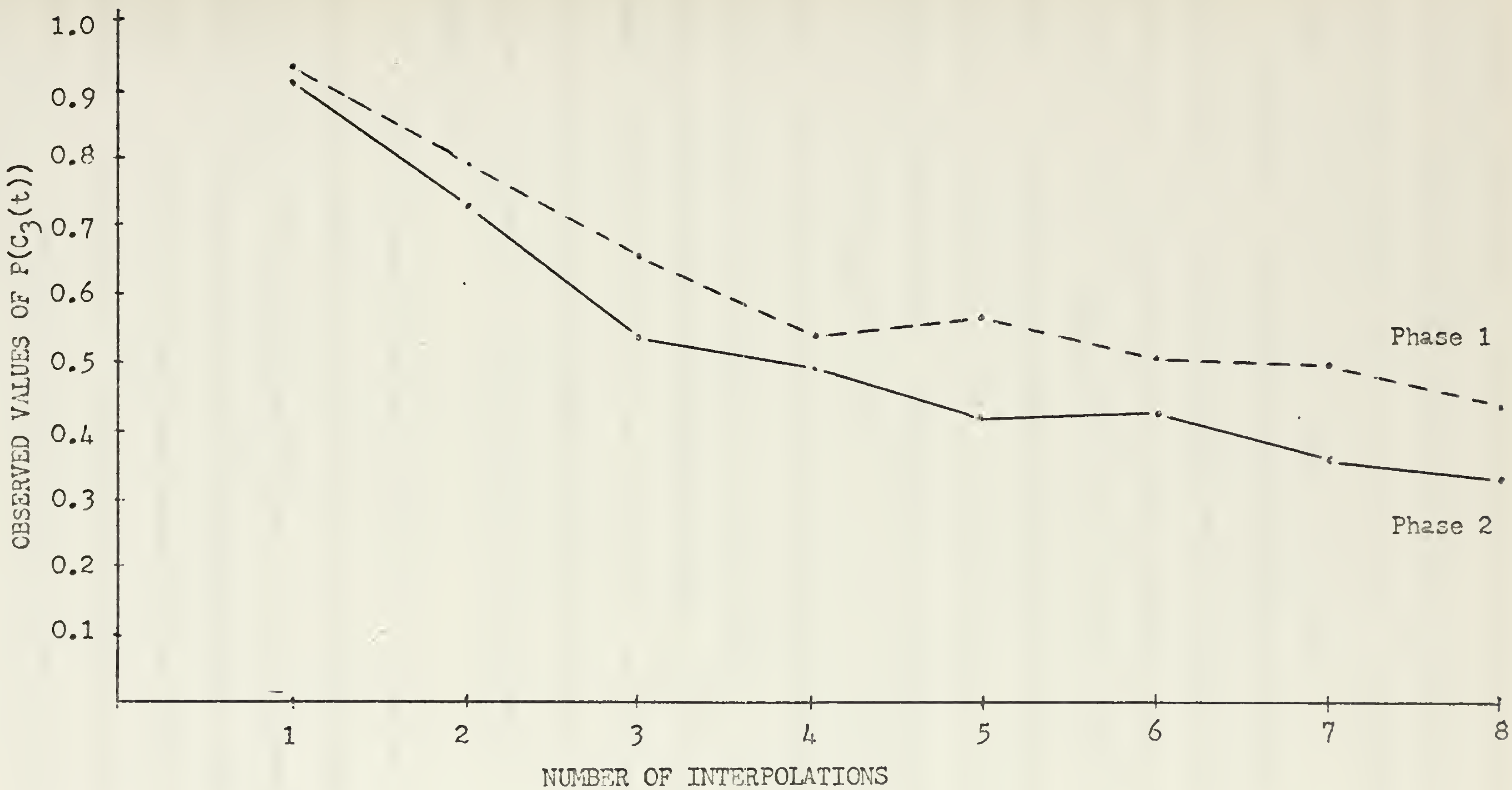

Figure 9. OBSERVED VALUES OF $P(C_3(t))$.
Phase 1 versus Phase 2.

probability distribution in modeling human short-term memory (Appendices A and B). Note that eight degrees of freedom were used in the chi-square tests because "fixing" one of the levels of t, interpolations, does not enable determination of the values of the other t's.

The values of the Percent Correct Responses for the N = 1 bit condition for phase 1 is closely aligned with the Percent Correct Responses for the N = 1 bit condition for phase 2 (Figure 5), but the values of the Percent Correct Responses for the N = 3 bits condition for phase 1 seems to run consistently higher than its phase 2 counterpart (Figure 5). Moreover, the magnitude of this difference appears to be related to the number of interpolations. The appropriate analysis was performed to test for phase and interpolation effects for the N = 3 bits condition. The design of the analysis was a 6 x 2 x 8 factorial with subjects a random factor and phase and interpolations fixed factors. The difference between the Percent Correct Responses for the N = 3 bits condition for phase 1, using discrimination net principles, and for phase 2, disregarding discrimination net principles resulted in an F value of 116.05 with degrees of freedom of 1 and 5, yielding a significance of < 0.001 (Appendix C).

The data collected from the conduct of the experiment is as would be expected in reference to the Percent Correct Responses as a function of the number of interpolations (Figure 5). The Percent Correct Responses for the N = 3 bits condition for both phases shows a trend toward degeneration as a function of increased interpolations (Figure 9). The significance of the number of interpolations resulted in an F value of 70.27. With degrees of freedom of 7 and 35, this yielded a significance of < 0.001 (Appendix C).

Further, an F value of 2.98 was obtained for the effects of the use or non-use of discrimination net principles and the number of interpolations involved in the experiment. The F value of 2.98 with 7 and 35 degrees of freedom was significant at the 0.025 level (Appendix C).

The effect of the number of trials upon the subjects' performance (Figure 10) was analyzed through implementation of trend analysis of trial means, summed over all N = 3 bits conditions for phases 1 and 2. This phase of the analysis yielded an F value of 2.44 with 9 and 45 degrees of freedom. This value is significant at the 0.025 level (Appendix D), indicating that trials had an effect on the subjects' performance. Further trend analysis was conducted in an effort to determine the linearity or curvature of the trial data. The sum of squares was partitioned into a sum of squares for linear regression, the linear component of the trend, and a sum of squares for curvature, the quadratic, cubic, quartic, and quintic components of the trend. The sum of squares for the components were:

LINEAR = 36.81

QUADRATIC = 15.001

CUBIC = 0.171

QUARTIC = 55.85

QUINTIC = 3.78.

The effect of the various components of the trend are computed in Appendix D.

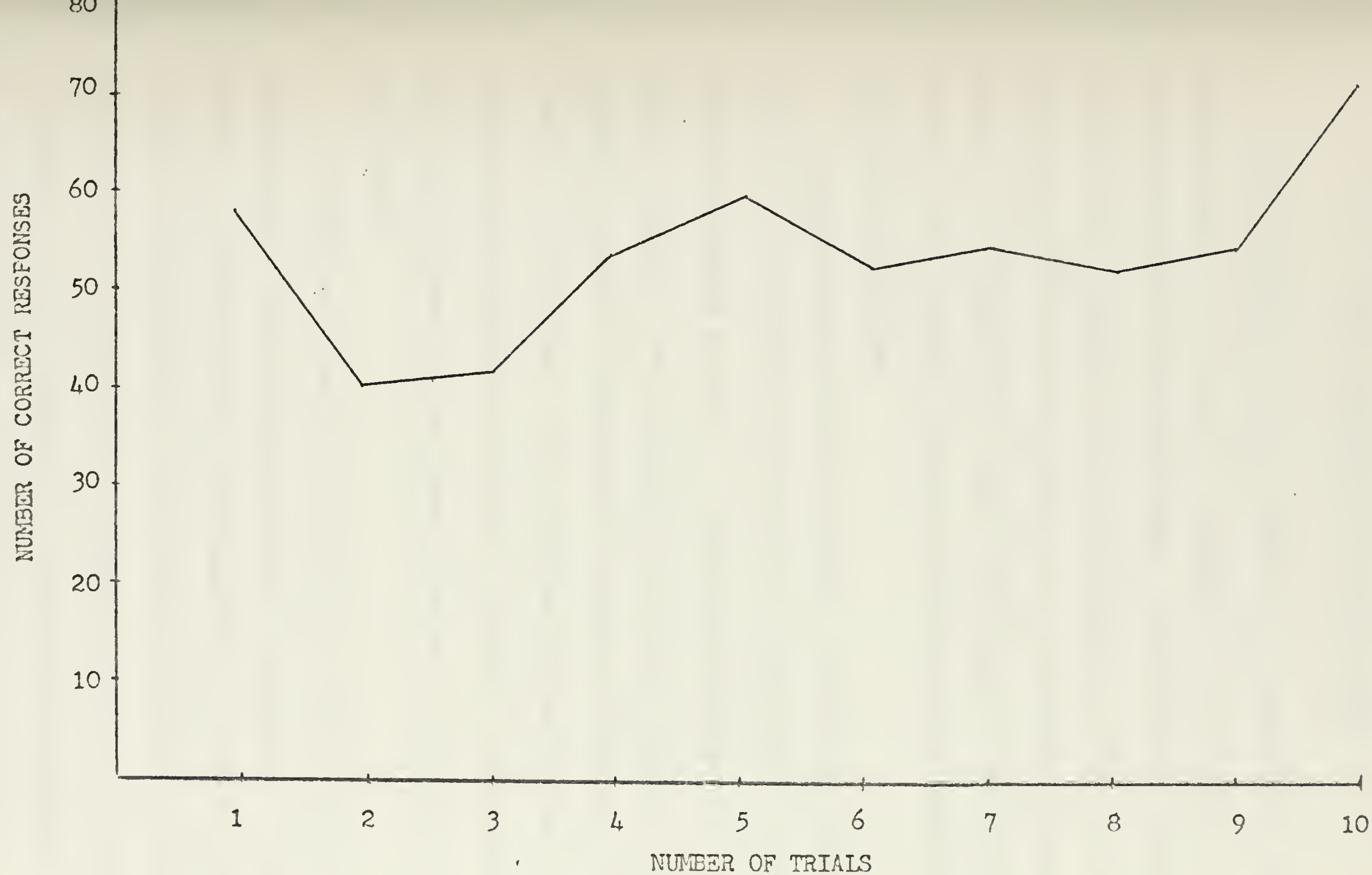

Figure 10. NUMBER OF CORRECT RESPONSES VERSUS NUMBER OF TRIALS. For Phase 1 and Phase 2.

III. DISCUSSION AND CONCLUSIONS

Taken separately, the two phases of the experiment tested the applicability of the binomial probability distribution as a means of modeling the human short-term memory. In addition, by comparing the results of the experiment by each independent variable, it was concluded that some insight would be gained as to the effect of the use or disregard of discrimination net theory principles, the number of interpolations, and the number of trials upon the subjects' performance in short-term memory exercises.

From the results of the experiment, it has been demonstrated that the binomial probability distribution is applicable in modeling human short-term memory in phase 1, using discrimination net principles. However, in phase 2 when discrimination net principles were disregarded, the binomial probability distribution failed the chi-square goodness-of-fit test at the 0.05 level. This dichotomy between success in the chi-square test in phase 1 and failure in phase 2 is attributed to be basic difference between the two phases, the use of discrimination net theory principles.

In reference to the discrimination net theory aspect of the experiment and how it influenced the subjects' performance, it may be stated that the trend toward memory degeneration as a function of increased number of interpolations is more pronounced in phase 2 which disregarded discrimination net principles (Figure 5 and Table 1). This contention is supported by the analysis of variance tests conducted. From these analyses, it may be concluded (Appendices C and D) that:

1. The difference in phases, use versus disregard of discrimination net principles, had a significant effect on the amount of information retained.

2. The number of interpolations involved in the experiment had a significant effect on the amount of information retained.

3. The effects of the discrimination net and number of interpolations interacted to a significant degree.

4. The number of trials undergone by the subjects had a significant effect on the amount of information retained.

5. The trend analysis of the effect of the number of trials leads to the conclusion that the trend of the trial means has linear, quadratic, cubic, quartic, and quintic components, of which the quartic is most prevalent (Appendix D and Figure 10).

The theoretical values of $P(C_3(t))$ for phase 1 and phase 2 were not compared because they were so closely aligned. This is as would be expected since these values are based on the Percent Correct Responses for the N = 1 bit (2 alternative) situation. The Percent Correct Responses for the N = 1 bit circumstance were quite similar in both phases since there were only two possible alternatives in this situation -- a situation wherein the discrimination net theory aspect of the experiment was not prevalent.

A possible problem associated with the chi-square tests was considered and somewhat reconciled. This was the problem of interclass dependence. This would be reflected in a subject-to-subject breakdown of the Percent Correct Responses at each level of t, interpolations. If some subjects seem to score well, while others seem to score poorly, then interclass dependence may be a problem. Such a breakdown was conducted and it seemed that a relatively good mix of successes and failures was distributed throughout the data (Appendix E).

The degree to which learning played a part in this experiment could have been minimized if differing sets of stimuli were used. As the experiment was conducted, the subjects were exposed to the same two sets of stimuli every other run of 10 trials. Although an effort was made to retard the learning effect by introducing a time span between successive tests, some degree of learning is inevitable as the experiment was conducted. A suggestion would be to use a different set of stimuli for each trial; although 80 sets of 10 stimuli seems impractical. It is interesting to note from Figure 10 that the subjects seem to start at a relatively high number of correct responses at trial 1, waver from trials 2 through 9 showing a trend toward an increased number of correct responses as a function of increased trials, peaking at trial 10. This phenomenon may be attributed to learning and the end-spurt effect.

The use of double throw toggle switches enabled the circuitry of the experiment, but may have detracted from the subjects' concentration since the switches were quite difficult to shift. Perhaps the use of push button switches which would not have in themselves distracted the subjects from the task, would have been more appropiate.

A data base of 60 trials was collected for each level of t interpolations, but it is obvious that an increased data base would have yielded more valid results. It would seem more logical to increase the data base by increasing the number of subjects rather than by increasing the number of trials per subject, thus obtaining a more varied cross-section of the population.

In summation, although the results of the experiment do not offer substantial evidence for the contention that the proposed model is applicable in modeling the human short-term memory, it does suggest

that further refinement of the model or proposals for similar models based on current theories in the field of human memory may prove to be valuable in the future.

APPENDIX A. CHI-SQUARE GOODNESS OF FIT TEST (PHASE 1)

1. H_0: the observed distribution is drawn from a population that is conditionally binomially distributed

2. H_1: the observed distribution is drawn from a population that is not conditionally binomially distributed

3. An α = P(reject H_0: H_0 true) of 0.05 (one-tail test) requires a value of 15.507, with eight degrees of freedom

4. Criterion: reject H_0 (accept H_1) if $\chi^2_8 > 15.507$; accept H_0 if $\chi^2_8 \leq 15.507$; where,

$$\chi^2_8 = \sum_{i=1}^{2} \sum_{j=1}^{8} \frac{(f_{oij} - f_{eij})^2}{f_{eij}}$$

5. Using the sample data, χ^2_8 values are tabulated for N = 3 bits.

interpolation	f_{oi}	f_{ei}	f_{oj}	f_{ej}	$\frac{(f_{oi}-f_{ei})^2}{f_{ei}}$	$\frac{(f_{oj}-f_{ej})^2}{f_{ej}}$
1	55	51.42	5	8.58	0.249	1.493
2	47	41.34	13	18.66	0.774	1.714
3	39	36.84	21	23.16	0.148	0.236
4	32	34.74	28	25.26	0.216	0.297
5	33	34.74	27	25.26	0.087	0.119
6	30	30.72	30	29.28	0.016	0.017
7	29	28.86	31	31.14	0.001	0.001
8	26	28.86	34	31.14	0.283	0.262
				$\Sigma\Sigma$ =	1.774 +	4.139

$$^2_8 = 5.913$$

Since $\chi^2_8(5.913) \leq 15.507$, accept H_0 and reject H_1. The sample distribution is concluded to be drawn from a conditionally binomial distribution.

APPENDIX B. CHI-SQUARE GOODNESS OF FIT TEST (PHASE 2)

1. H_0: the observed distribution is drawn from a population that is conditionally binomially distributed

2. H_1: the observed distribution is drawn from a population that is not conditionally binomially distributed

3. An α = P(reject H_0: H_0 true) of 0.05 (one-tail test) requires a value of 15.507, with eight degrees of freedom

4. Criterion: reject H_0 (accept H_1) if $\chi^2_8 > 15.507$; accept H_0 if $\chi^2_8 \leq 15.507$; where,

$$\chi^2_8 = \sum_{i=1}^{2} \sum_{j=1}^{8} \frac{(f_{oij}-f_{eij})^2}{f_{eij}}$$

5. Using the sample data, χ^2_8 values are tabulated for N = 3 bits.

interpolation	f_{oi}	f_{ei}	f_{oj}	f_{ej}	$\frac{(f_{oi}-f_{ei})^2}{f_{ei}}$	$\frac{(f_{oj}-f_{ej})^2}{f_{ej}}$
1	54	51.42	6	8.58	0.129	0.776
2	43	41.34	17	18.66	0.066	0.147
3	32	43.74	28	16.26	3.151	8.476
4	29	34.74	31	25.26	0.948	1.304
5	25	36.84	35	23.16	3.805	6.052
6	22	30.72	38	29.28	2.475	2.691
7	21	28.86	39	31.14	2.140	1.984
8	19	27.06	41	32.94	2.400	1.974
					$\Sigma\Sigma$ =13.149 +	23.404

$${}^{2}_{8} = 36.553$$

Since χ^2_8 (36.553) > 15.507, reject H_0 and accept H_1. The sample distribution is concluded to be drawn from a distribution that is not conditionally binomially distributed.

APPENDIX C. TREATMENTS-BY-TREATMENTS-BY-SUBJECTS ANALYSIS OF VARIANCE

Subject	PHASE 1 interpolation								PHASE 2 interpolation								Sum
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	
1	10	7	7	6	6	5	5	4	9	7	6	5	5	4	3	3	92
2	9	8	6	4	5	6	5	4	10	7	5	5	4	3	3	3	87
3	9	8	7	5	5	4	4	4	9	7	5	5	4	3	3	3	85
4	8	7	7	6	6	5	5	5	8	8	5	4	4	4	4	3	89
5	10	9	6	7	7	5	5	4	9	7	6	5	4	4	4	4	96
6	9	8	6	4	4	5	5	5	9	7	5	5	4	4	4	3	87
Sum	55	47	39	32	33	30	29	26	54	43	32	29	25	22	21	19	536

interpolation

subject	1	2	3	4	5	6	7	8
1	19	14	13	11	11	9	8	7
2	19	15	11	9	9	9	8	7
3	18	15	12	10	9	7	7	7
4	16	15	12	10	10	9	9	8
5	19	16	12	12	11	9	9	9
6	18	15	11	9	8	9	9	8
	109	90	71	61	58	52	50	46

subject	PHASE 1	PHASE 2
1	50	42
2	47	40
3	46	39
4	49	40
5	53	43
6	46	41
	291	245

$$SS_{\text{subjects}} = \frac{(92)^2}{16} + \frac{(87)^2}{16} + \ldots + \frac{(87)^2}{16} - \frac{(536)^2}{96} = 5.09$$

$$SS_{\text{phase}} = \frac{(291)^2}{48} + \frac{(245)^2}{48} - \frac{(536)^2}{96} = 22.05$$

$$SS_{\text{interpolation}} = \frac{(109)^2}{12} + \frac{(90)^2}{12} + \ldots + \frac{(45)^2}{12} - \frac{(536)^2}{96} = 285.34$$

$$SS_{\text{phase} \cdot \text{interpolation}} = \frac{(55)^2}{6} + \frac{(47)^2}{6} + \ldots + \frac{(19)^2}{6} - \frac{(536)^2}{96} - SS_{\text{phase}} - SS_{\text{interpolation}} = 4.28$$

$$SS_{\text{error for phase}} = \frac{(50)^2}{8} + \frac{(47)^2}{8} + \ldots + \frac{(41)^2}{8} - \frac{(536)^2}{96} - SS_{\text{subjects}} - SS_{\text{phase}} = 0.9511$$

$$SS_{\text{error for interpolation}} = \frac{(19)^2}{2} + \frac{(19)^2}{2} + \ldots + \frac{(8)^2}{2} - \frac{(536)^2}{96} - SS_{\text{subjects}} - SS_{\text{interpolation}} = 20.41$$

$$SS_{\text{error for phase} \cdot \text{inter.}} = (10)^2 + (7)^2 + \ldots + (3)^2 - \frac{(536)^2}{96} - SS_{\text{subjects}} - SS_{\text{phase}} - SS_{\text{inter.}} - SS_{\text{phase} \cdot \text{inter.}} - SS_{\text{error for phase}} - SS_{\text{error for inter.}} = 7.17$$

SOURCE	SS	df	MEAN SQUARE	F
TOTAL	345.29	95	-	-
Subjects	5.09	5	-	-
phase	22.05	1	22.05	116.05
interpolation	285.34	7	40.76	70.27
phase · inter.	4.28	7	0.61	2.98
error phase	0.95	5	0.19	-
error inter.	20.41	35	0.58	-
error phase·inter.	7.17	35	0.21	-

An F value of 116.05 with 1 and 5 degrees of freedom is significant at the <0.001 level. This level of significance is associated with the independent variable, phase (use or non-use of discrimination net principles).

An F value of 70.27 with 7 and 35 degrees of freedom is significant at the 0.001 level. This level of significance is associated with the independent variable, number of interpolations.

An F value of 2.98 with 7 and 35 degrees of freedom is significant at the 0.025 level. This level of significance is associated with the interaction between the independent variables, phase and number of interpolations.

APPENDIX D. TREND ANALYSIS

	trials										
	1	2	3	4	5	6	7	8	9	10	sum
subjects											
1	10	5	10	8	10	5	13	8	11	12	92
2	12	5	6	11	11	8	8	8	8	10	87
3	10	4	7	9	11	6	11	10	4	13	85
4	10	8	6	8	7	10	10	8	8	14	89
5	9	8	9	8	11	10	8	9	11	13	96
6	7	10	4	9	9	13	4	9	12	10	87
sum	58	40	42	53	59	52	54	52	54	72	536

$$SS_{total} = (10)^2 + (12)^2 + \ldots + (72)^2 - \frac{(536)^2}{60} = 356$$

$$SS_{trial} = \frac{(58)^2}{6} + \frac{(40)^2}{6} + \ldots + \frac{(72)^2}{6} - \frac{(536)^2}{60} = 119$$

$$SS_{subject} = \frac{(92)^2}{10} + \frac{(87)^2}{10} + \ldots + \frac{(87)^2}{10} - \frac{(536)^2}{60} = 8$$

$$SS_{subject \cdot trial} = SS_{total} - SS_{subject} - SS_{trial} = 229$$

SOURCE	SS	df	MEAN SQUARE	F
trials	119	9	13.2	2.58
subjects	8	5	1.6	-
subjects · trials	229	45	5.1	-
TOTAL	356	59	19.9	

An F value of 2.58 with 9 and 45 degrees of freedom is significant at the 0.025 level.

The sum of squares for trials may be partitioned into a sum of squares for linear regression, the linear component of the trend, and a sum of squares for curvature, the quadratic component of the trend. With k = 10 trials, the orthogonal coefficients for the linear component of the trend of the trial means are -9, -7, -5, -3, -1, 1, 3, 5, 7, 9. The sum of squares for the linear component of the trend of the trial means is given by,

$$\text{Linear} = \frac{D_1^2}{n\Sigma a_1^2}$$

where,

n = the number of subjects

$\Sigma a_1^2 = \Sigma$ (orthogonal coefficients)^2

$$\text{Linear} = \frac{(-9(58) - 7(40) - \ . \ . \ . + 9(72))^2}{6(330)}$$
$$= 36.81$$

An F value of 7.22 (36.81/5.1) with 9 and 45 degrees of freedom is significant at the < 0.001 level.

Since this value of the sum of squares is less than the value of the sum of squares for trials, the sum of squares for curvature must be solved for. With k = 10 trials, the orthogonal coefficients of the trend of the trial means for the quadratic component are 6, 2, -1, -3, -4, -4, -3, -1, 2, 6. The sum of squares for the quadratic component of the trend of the trial means is given by,

$$\text{Quadratic} = \frac{D_2^2}{n\Sigma a_2^2}$$

or,

$$\text{Quadratic} = \frac{(6(58) + 2(40) + \ . \ . \ . + (72))^2}{6(132)}$$
$$= 15.001$$

An F value of 2.94 with 9 and 45 degrees of freedom is significant at the 0.025 level.

Further, with k = 10 trials, the orthogonal coefficients of the trend of the trial means for the cubic component are -42, 14, 35, 31, 12, -12, -31, -35, -14, 42. The sum of squares for the cubic component of the trend of trial means is given by,

$$\text{CUBIC} = \frac{D_3^2}{n\Sigma a_3^2}$$

or,

$$\text{Cubic} = \frac{(-42(58) + 14(40) + \ . \ . \ . + 42(72))^2}{6(8580)}$$
$$= 0.171$$

An F value of 0.033 with 9 and 45 degrees of freedom is significant at the > 0.250 level.

Further, the orthogonal coefficients for the quartic component of the trend of the trial means are 18, -22, -17, 3, 18, 18, 3, -17, -22, 18. Utilizing the formula for the sum of squares for the quartic component,

$$\text{QUARTIC} = \frac{D_4^2}{n\Sigma a_4^2}$$

or,

$$\text{Quartic} = \frac{(18(58) - 22(40) + \ . \ . \ . + 18(72))^2}{6(2860)}$$
$$= 55.85$$

An F value of 10.95 with 9 and 45 degrees of freedom is significant at the < 0.001 level.

Finally, the orthogonal coefficients for the quintic component of the trend of the trial means are -6, 14, -1, -11, -6, 6, 11, 1, -14, 6. The sum of squares for the quintic component is,

$$\text{QUINTIC} = \frac{D_5^2}{n\Sigma a_5^2}$$

or,

$$\text{Quintic} = \frac{(-6(58) + 14(40) + \ . \ . \ . + 6(72))^2}{6(780)}$$
$$= 3.78$$

An F value of 0.74 with 9 and 45 degrees of freedom is significant at the > 0.25 level.

Thus, the trend of the sum of squares for the trial means is seen to contain linear, quadratic, cubic, quartic, and quintic components, with the quartic component having the most significance and the cubic component having the least significance of trend.

APPENDIX E. RAW DATA

Subject 1, Phase 1

Interpolation	Trial 1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
2	1	0	0	1	1	0	1	1	1	1
3	1	1	0	1	1	0	0	1	1	1
4	1	0	1	0	1	1	1	0	1	0
5	1	0	1	0	1	0	1	1	0	1
6	1	0	1	0	1	0	1	0	0	1
7	1	0	1	0	1	0	1	0	1	0
8	1	0	0	1	0	1	0	0	0	1

Subject 1, Phase 2

Interpolation	Trial 1	2	3	4	5	6	7	8	9	10
1	1	0	1	1	1	1	1	1	1	1
2	1	0	1	1	0	0	1	1	1	1
3	0	1	1	0	1	0	1	0	1	1
4	0	1	0	1	0	0	1	0	1	1
5	0	1	0	1	0	1	1	0	1	0
6	0	0	1	0	0	0	1	1	0	1
7	0	0	0	0	0	0	1	0	1	1
8	0	0	1	0	1	0	0	1	0	1

1 = Success

0 = Failure

Subject 2, Phase 1

	Trial									
	1	2	3	4	5	6	7	8	9	10
Interpolation										
1	1	1	1	1	1	1	1	1	0	1
2	1	1	1	0	1	1	1	0	1	1
3	1	0	1	0	1	1	0	1	0	1
4	0	0	0	1	1	1	0	1	0	0
5	1	0	0	1	1	0	1	0	0	1
6	1	0	0	1	1	0	0	1	1	1
7	1	0	0	1	1	0	0	0	1	1
8	1	0	0	1	0	1	0	0	1	0

Subject 2, Phase 2

	Trial									
	1	2	3	4	5	6	7	8	9	10
Interpolation										
1	1	1	1	1	1	1	1	1	1	1
2	1	1	0	1	1	0	1	0	1	1
3	1	0	0	1	0	1	0	1	1	0
4	0	1	0	0	1	0	1	1	0	1
5	0	0	1	1	0	0	1	0	1	0
6	1	0	0	0	1	0	0	1	0	0
7	1	0	0	1	0	1	0	0	0	0
8	0	0	1	0	0	0	1	0	0	1

Subject 3, Phase 1

	Trial									
	1	2	3	4	5	6	7	8	9	10
Interpolation										
1	1	1	0	1	1	1	1	1	1	1
2	1	0	1	1	1	0	1	1	1	1
3	1	0	1	1	0	1	1	1	0	1
4	1	0	0	1	0	1	0	1	0	1
5	1	0	1	0	1	0	1	1	0	0
6	0	1	0	0	1	0	1	0	0	1
7	0	0	1	0	1	0	1	0	0	1
8	0	0	1	0	0	1	0	1	1	0

Subject 3, Phase 2

	Trial									
	1	2	3	4	5	6	7	8	9	10
Interpolation										
1	1	1	0	1	1	1	1	1	1	1
2	1	0	1	0	1	1	1	1	0	1
3	1	0	1	0	1	0	0	1	0	1
4	1	0	0	1	1	0	1	0	0	1
5	0	1	0	1	0	0	1	0	0	1
6	0	0	0	1	0	0	1	0	0	1
7	0	0	0	1	1	0	0	1	0	0
8	1	0	0	0	1	0	0	0	0	1

Subject 4, Phase 1

	Trial									
	1	2	3	4	5	6	7	8	9	10
Interpolation										
1	1	0	1	1	0	1	1	1	1	1
2	1	1	0	1	0	1	1	1	0	1
3	1	1	0	1	1	0	1	0	1	1
4	1	0	1	0	1	1	0	0	1	1
5	0	1	1	0	1	1	1	0	0	1
6	0	0	1	0	1	0	1	0	1	1
7	0	1	1	0	0	1	0	1	0	1
8	0	1	0	1	0	0	1	1	0	1

Subject 4, Phase 2

	Trial									
	1	2	3	4	5	6	7	8	9	10
Interpolation										
1	1	1	0	1	0	1	1	1	1	1
2	1	1	0	1	1	0	1	1	1	1
3	1	0	0	1	0	1	1	0	0	1
4	0	0	0	1	0	1	0	0	1	1
5	1	0	1	0	0	1	0	0	0	1
6	1	1	0	0	0	1	0	0	1	0
7	1	0	0	0	1	0	0	1	0	1
8	0	0	0	0	1	0	1	1	0	0

Subject 5, Phase 1

	Trial									
	1	2	3	4	5	6	7	8	9	10
Interpolation										
1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	0	1	1
3	1	0	1	0	1	1	0	0	1	1
4	1	0	1	1	0	1	1	0	1	1
5	1	1	0	1	1	0	1	0	1	1
6	0	1	0	0	0	0	1	1	1	1
7	1	0	0	1	1	1	0	1	0	0
8	1	0	0	0	1	0	1	0	1	0

Subject 5, Phase 2

	Trial									
	1	2	3	4	5	6	7	8	9	10
Interpolation										
1	1	1	1	1	1	1	1	0	1	1
2	1	1	0	1	1	1	0	1	1	0
3	0	1	1	0	0	1	0	1	1	1
4	0	1	1	0	0	1	0	1	0	1
5	0	0	1	0	1	0	0	1	0	1
6	0	0	0	0	1	0	1	0	1	1
7	0	0	1	0	1	0	0	1	0	1
8	0	0	0	1	0	1	0	1	0	1

Subject 6, Phase 1

	Trial									
	1	2	3	4	5	6	7	8	9	10
Interpolation										
1	1	0	1	1	1	1	1	1	1	1
2	1	1	0	1	1	1	0	1	1	1
3	0	1	0	1	1	0	1	1	0	1
4	0	1	0	0	1	1	0	0	0	1
5	0	1	0	0	1	1	0	1	0	0
6	1	0	0	1	0	1	0	0	1	1
7	0	1	0	1	1	0	0	1	1	0
8	0	1	1	0	0	1	0	0	1	1

Subject 6, Phase 2

	Trial									
	1	2	3	4	5	6	7	8	9	10
Interpolation										
1	1	0	1	1	1	1	1	1	1	1
2	0	1	1	1	1	0	1	0	1	1
3	1	0	0	1	0	1	0	0	1	1
4	1	0	0	1	1	1	0	0	1	0
5	0	1	0	0	0	1	0	1	1	0
6	1	0	0	0	0	1	0	1	1	0
7	0	1	0	0	0	1	0	1	1	0
8	0	1	0	0	0	1	0	0	0	1

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(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

Application of the Binomial Probability Distribution and Discrimination Net Theory in Modeling Short-Term Human Memory in a Console Operator Situation

4. DESCRIPTIVE NOTES (Type of report and, inclusive dates)

Master's Thesis; June 1973

5. AUTHOR(S) (First name, middle initial, last name)

Eugene Bal III

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
June 1973	68	21

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

Recall of paired-associate, binary-coded, three-digit numbers was tested in an effort to determine the applicability of the binomial probability distribution in modeling short-term, human memory. Further, the use of discrimination net theory principles in tests of recall was examined. Although the analysis showed the binomial probability distribution to be applicable in modeling memory to the 0.95 level of significance, by no means is the model capable of universal application; in fact, its application is probably as localized as the assumptions made about it and its functional frame of reference - that of a console operator.

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Bionomial Probability Distribution						
Discrimination Net Theory						
Console Operator Situation						

DD FORM 1 NOV 65 1473 (BACK)

S/N 0101-807-6821

Security Classification

A-31409

Thesis 144175
B1745 Bal
c.1 Application of the binomial probability distribution and discrimination net theory in modeling short-term human memory in a console operator situation.